

# 1 Preliminary External Validation of $\lambda_F$ for SPX

We conducted a preliminary external validation study for the SPX branch of the  $\lambda_F$  model using hourly data and event-based evaluation windows of 24h, 48h, and 72h. External stress events were defined independently from  $\lambda_F$  through a combination of realized volatility spikes, drawdown conditions, VIX shock behavior, and Treasury yield shock behavior. This framework was designed to test whether elevated  $\lambda_F$  states precede externally observable market stress rather than merely reflecting the model's own internal distribution.

At the aggregate level, the SPX branch of  $\lambda_F$  showed its strongest predictive behavior in the 48h–72h horizon. The observed event lift was modest at 24h ( $1.06\times$ ), but increased materially at 48h ( $1.48\times$ ) and remained elevated at 72h ( $1.50\times$ ). Median lead times also suggested that successful alerts tend to precede external stress by roughly one day, supporting the interpretation that SPX  $\lambda_F$  behaves more like a short-horizon stress field indicator than an immediate shock detector.

A more detailed decomposition revealed that predictive strength is not homogeneous across alert types. When alerts were separated by level and dominant component, two distinct sub-regimes emerged:

- **Minor + lev\_stress-dominant:** These alerts produced their strongest signal in the 48h–72h range, reaching a  $1.67\times$  lift at 72h. This pattern is consistent with an interpretation of these alerts as an *early tension regime*, where macro-financial pressure is building before fully manifesting as an external stress event.
- **Critical + fng-dominant:** This was the strongest overall sub-regime. Although this group was relatively small in count, it produced the highest lift values across the matrix, including  $1.88\times$  at 24h,  $2.30\times$  at 48h, and  $1.86\times$  at 72h.

This suggests that when the SPX branch reaches a critical state under sentiment-dominant conditions, the resulting alert may represent a high-conviction short-term stress signal rather than a broad background warning.

These findings imply that the SPX implementation of  $\lambda_F$  may contain at least two operationally distinct dynamics: a slower macro-pressure pathway captured by *lev\_stress-dominant minor* alerts, and a sharper short-term stress pathway captured by *fng-dominant critical* alerts. While the present sample remains limited and should therefore be treated as preliminary, the results provide evidence that the SPX branch of  $\lambda_F$  is not informationally uniform and that its predictive value increases substantially when alert level and dominant component are considered jointly.